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10MAT41

Fourth Semester B.E. Degree Examination, December 2012

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Using the Taylor's series method, solve the initial value problem $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$ **(06 Marks)**

b. Employ the fourth order Runge-Kutta method to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ at the points $x = 0.2$ and $x = 0.4$. Take $h = 0.2$. **(07 Marks)**

c. Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$. Find $y(0.4)$ using the Milne's predictor-corrector method. Apply the corrector formula twice. **(07 Marks)**

2 a. Employing the Picard's method, obtain the second order approximate solution of the following problem at $x = 0.2$.
$$\frac{dy}{dx} = x + yz, \quad \frac{dz}{dx} = y + zx, \quad y(0) = 1, \quad z(0) = -1.$$ **(06 Marks)**

b. Using the Runge-Kutta method, find the solution at $x = 0.1$ of the differential equation $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$ under the conditions $y(0) = 1$, $y'(0) = 0$. Take step length $h = 0.1$. **(07 Marks)**

c. Using the Milne's method, obtain an approximate solution at the point $x = 0.4$ of the problem $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$, $y(0) = 1$, $y'(0) = 0.1$. Given that $y(0.1) = 1.03995$, $y(0.2) = 1.138036$, $y(0.3) = 1.29865$, $y'(0.1) = 0.6955$, $y'(0.2) = 1.258$, $y'(0.3) = 1.873$. **(07 Marks)**

3 a. If $f(z) = u + iv$ is an analytic function, then prove that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$. **(06 Marks)**

b. Find an analytic function whose imaginary part is $v = e^x\{(x^2 - y^2)\cos y - 2xy\sin y\}$. **(07 Marks)**

c. If $f(z) = u(r, \theta) + iv(r, \theta)$ is an analytic function, show that u and v satisfy the equation $\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0$. **(07 Marks)**

4 a. Find the bilinear transformation that maps the points 1, i, –1 onto the points i, 0, –i respectively. **(06 Marks)**

b. Discuss the transformation $W = e^z$. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)}dz$, where C is the circle $|z| = 3$. **(07 Marks)**

PART – B

5 a. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(06 Marks)**

b. Obtain the series solution of Bessel's differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ in the form $y = AJ_n(x) + BJ_{-n}(x)$. **(07 Marks)**

c. Derive Rodrique's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. **(07 Marks)**

6 a. State the axioms of probability. For any two events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(06 Marks)**

b. A bag contains 10 white balls and 3 red balls while another bag contains 3 white balls and 5 red balls. Two balls are drawn at ransom from the first bag and put in the second bag and then a ball is drawn at random from the second bag. What is the probability that it is a white ball? **(07 Marks)**

c. In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25% 40% of the total production. Out of these 5%, 4%, 3% and 2% respectively are defective. A bolt is drawn at random from the production and is found to be defective. Find the probability that it was manufactured by A or D. **(07 Marks)**

7 a. The probability distribution of a finite random variable X is given by the following table:

x_i	-2	-1	0	1	2	3
$p(x_i)$	0.1	k	0.2	2k	0.3	k

Determine the value of k and find the mean, variance and standard deviation. **(06 Marks)**

b. The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that (i) exactly 2 will be defective, (ii) at least 2 will be defective, (iii) none will be defective. **(07 Marks)**

c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation, given that A(0.5) = 0.19 and A(1.4) = 0.42, where A(z) is the area under the standard normal curve from 0 to z >0. **(07 Marks)**

8 a. A biased coin is tossed 500 times and head turns up 120 times. Find the 95% confidence limits for the proportion of heads turning up in infinitely many tosses. (Given that $z_c = 1.96$) **(06 Marks)**

b. A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure:

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 (in appropriate unit)

Can it be concluded that, on the whole, the stimulus will change the blood pressure. Use $t_{0.05}(11) = 2.201$. **(07 Marks)**

c. A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given by the following table:

x	1	2	3	4	5	6
Frequency	15	6	4	7	11	17

Test the hypothesis that the die is unbiased.
(Given that $\chi^2_{0.05}(5) = 11.07$ and $\chi^2_{0.01}(5) = 15.09$) **(07 Marks)**

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Fourth Semester B.E. Degree Examination, December 2012
Concrete Technology

Time: 3 hrs.   Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting atleast TWO questions from each part.***
2. Use of IS – 10262 – 2009 is permitted.

PART – A

1 a. What are the chemical compositions of cement? How the cement is manufactured give flow chart. **(10 Marks)**
b. Explain the terms normal consistence initial and final setting time. **(10 Marks)**

2 a. Explain the importance of size shape and texture of aggregate. **(10 Marks)**
b. Explain the role of admixture in concrete making. Write briefly about the accelerators and retardors. **(10 Marks)**

3 a. What is meant by workability of concrete? What are the factors affecting the workability of the concrete. **(10 Marks)**
b. Explain briefly the suggrigation and bleeding of concrete. **(10 Marks)**

4 a. Why curing is necessary. Explain briefly the different types of curing. **(10 Marks)**
b. What are the processes of manufacturing of the cement concrete? Explain briefly. **(10 Marks)**

PART – B

5 a. Explain the influence of water/cement ratio on the strength of the concrete. **(08 Marks)**
b. Explain how the compressive and tensile strength of the concrete is determined. **(08 Marks)**
c. What do you mean by non – destructive testing of concrete? Explain any one of them. **(04 Marks)**

6 a. What are the factors that influence the strength of concrete? **(06 Marks)**
b. Explain the different types of shrinkage that take place in concrete. **(06 Marks)**
c. Write a note on creep of concrete. **(08 Marks)**

7 a. What is meant by concrete – mix – designs? Write the steps involved in the method of mix – design (IS – 10262 – 2009) **(14 Marks)**
b. What are the factors reducing the durability of the concrete. **(06 Marks)**

8 Write short notes :
a. Light weight concrete
b. Mineral – admixtures
c. Fibre – reinforced concrete
d. Compaction of concrete. **(20 Marks)**

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10CV43

Fourth Semester B.E. Degree Examination, December 2012
Structural Analysis – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Distinguish between static and kinematic indeterminate structures, with example. **(04 Marks)**

b. Derive an expression for the strain energy stored in a member due to bending. **(06 Marks)**

c. A cantilevers truss is loaded as shown in Fig. Q1(c). The cross sectional area of the members is as indicated in the figure. Find the strain energy stored due to loading. Take E = 72 GPa. **(10 Marks)**

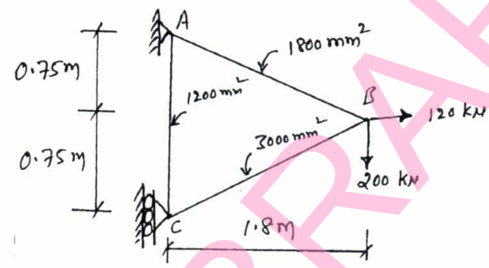

Fig. Q1(c)

2 a. A cantilever of length 4 m is loaded as shown in Fig. Q2(a). Taking EI constant for the whole length of the beam, calculate the deflection at the free end by moment – area method. **(08 Marks)**

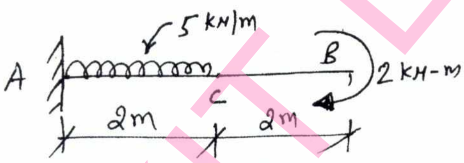

Fig. Q2(a)

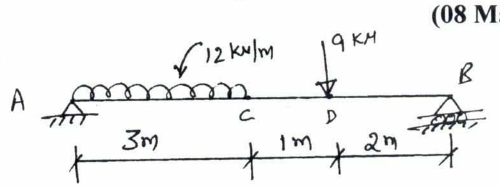

Fig. Q2(b)

b. Determine the slope at A and deflection at C in the beam loaded as shown in Fig. Q2(b), by conjugate beam method. Take EI constant. **(12 Marks)**

3 a. Find the deflection under concentrated load for the beam shown in Fig. Q3(a), using Castigliano's first theorem. Take $E = 2 \times 10^8$ kN/m^2 and $I = 14 \times 10^{-6}$ m^4 **(10 Marks)**

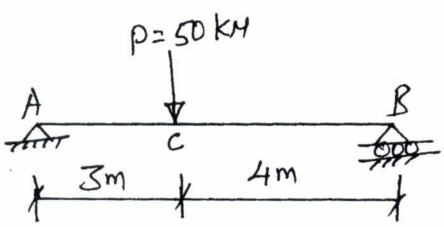

Fig. Q3(a)

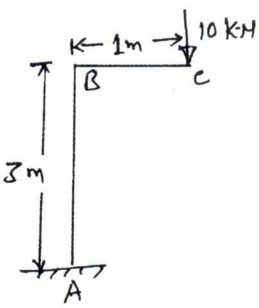

Fig. Q3(b)

b. Find the vertical deflection at C for the bent shown in Fig. Q3(b). Using strain energy method. Take EI constant. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

4 Find the vertical deflection at F for the pin jointed truss shown in Fig. Q4, using unit load method. Assume cross sectional area of each member as 1000 mm^2 and E = 200 GPa. **(20 Marks)**

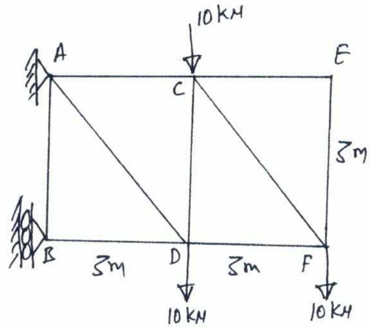

Fig. 4

PART – B

5 a. A three hinged parabolic arch hinged at the supports and at the crown has a span of 24 m and a central rise of 4m. It carries a concentrated load of 50 kN at 18 m from the left support and a udl of 30 kN/m over the left half portion. Determine the bonding moment, normal thrust and radial shear at a section 6 m from left support. **(12 Marks)**

b. A suspension cable having supports at same level has a span of 40 m and maximum dip of 4 m. The cable is loaded with udl of 10 kN/m through its length. Calculate the maximum and minimum tension in the cable. Also find the length of the cable. **(08 Marks)**

6 a. Draw BMD for the propped cantilever loaded as shown in Fig. Q6(a), by consistent deformation method. The supports at A and B remain at the same level after the load. **(10 Marks)**

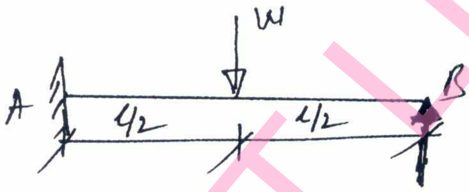

Fig. Q6(a)

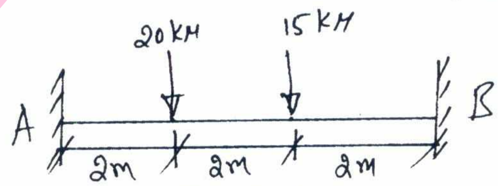

Fig. Q6(b)

b. A fixed beam is loaded as shown in Fig. Q6(b). Draw BMD. **(10 Marks)**

7 Analyse the continuous beam shown in Fig. Q7 by Clayperon's theorem of three moments. Draw BMD, SFD and elastic curve. EI constant throughout **(20 Marks)**

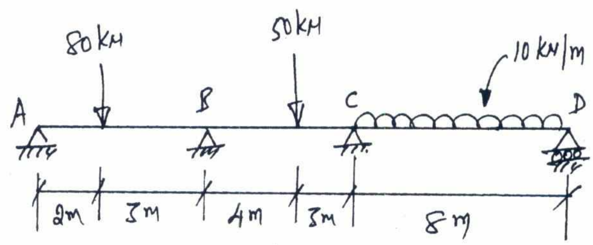

Fig. Q7

8 A parabolic arch hinged at the ends has a span of 30 m and rise of 5 m. A concentrated load of 12 kN acts at 10 m from the left hinge. The second moment of area varies as the secant of the slope of the rib axis. Calculate the horizontal thrust and the reactions at the hinges. Also calculate maximum bending moment anywhere on the arch. **(20 Marks)**

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Fourth Semester B.E. Degree Examination, December 2012
Surveying – II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Assume any missing data suitably.

PART – A

1 a. Describe the procedure of measuring horizontal angle with a transit. **(06 Marks)**
b. State what errors are eliminated by repetition method. **(04 Marks)**
c. Explain the prolongation of a straight line using theodolite:
i) Which is in adjustment ii) Which is in poor adjustment. **(10 Marks)**

2 a. What are the fundamental lines of a theodolite state the desired relationship between them. **(10 Marks)**
b. A dumpy level was kept midway between two points A & B 100m apart. The readings on staff held at A & B were 2.340 m and 1.795 m. The instrument was then kept at C, 20 m from A along BA produced. The readings on the staff held at A & B were 1.985 m and 1.435 m respectively. Calculate the correct readings at A & B with the instrument at C. Is the line of collimation inclined upward or downward? **(10 Marks)**

3 a. A theodolite was set up at a distance of 150 m from tower. The angle of elevation to the top of the tower was 10°8′ while the angle of depression to the foot of the tower was 3°12′. The staff reading on the B.M. of R.L. 50.217m with the telescope horizontal was 0.880 m. Find the height of the tower and the R.L. of the top of the parapet. **(06 Marks)**
b. What are the applications of total station? **(02 Marks)**
c. To find the elevation of the top(P) of a hill, a flag staff of height 1.5m was erected and the following observations were made from two stations A & B at considerably different elevations 156m apart. The angle of elevation from A to the top of the flag staff was 38°24′ and that from B to the same point 26°12′. A vane 1.2m above the foot of a staff held on A was sighted from B and the angle of elevation was observed to be 9°54′. The height of the instrument axis at A was 1.494 and the R.L. of the instrument axis at B was 45.00m. Find the horizontal distance P from B and the R.L. of P. **(12 Marks)**

4 a. Differentiate between fixed hair method and movable hair method. **(06 Marks)**
b. Following observations were taken with a tacheometer fitted with an anallactic lens having a value of constant to be 100 and staff held vertical.

Inst. Station	Staff station	R.B.	Vertical angle	Staff readings.
0	P	N 37° W	4° 12′	0.910, 1.510, 2.110
	Q	N 23° E	5° 42′	1.855, 2.705, 3.555

Determine the gradient between points P & Q. **(10 Marks)**
c. Two vertical angles to vanes fixed at 1m and 3m above the foot of the staff held vertically at a station A were +2°15′ and +5°50′ respectively. Find the horizontal distance and R.L. of A, if R.L. of instrument axis is 235.665 m above the datum. **(04 Marks)**

PART – B

5 a. Derive the expressions for the following elements of a simple curve.
i) External distance ii) Mid ordinate **(04 Marks)**

b. A curve has a radius of 400m and a deflection angle of 40°. The chainage of point of intersection 1804.25 m. Compute the necessary data to setout simple curve by offsets from chords produced. Take Peg interval as 20 m. **(08 Marks)**

c. Two straight lines with a total deflection angle of 72°30′ are to be connected by a compound curve of two branches of equal length. The radius of the first arc is 350 m and that of the second arc is 500m and chainage of vertex is 1525m. Find the chainages of two tangent points and that of point of compound curvature. **(08 Marks)**

6 a. Explain with neat sketches the different triangulation systems. **(06 Marks)**

b. Define the following :
i) Satellite station ii) Reduction to centre. **(04 Marks)**

c. From an eccentric station ‘S’ 9m to the west of the main station B, the following angles were measured ∠BSC = 33°15′40″ and ∠ASC = 67°56′20″. The stations S and C are to the opposite sides of the line AB. Calculate the correct angle ABC if the lengths AB & BC are 3,896.8 m and 4,470.72 m respectively. **(10 Marks)**

7 a. What is super elevation? Derive an expression for the amount of super elevation. **(06 Marks)**

b. With the help of neat sketches, explain types of vertical curves. **(06 Marks)**

c. A circular curve of radius 300m on a railway line gauge 1.5m is to be provided with transition curves at both ends. The super elevation is to be restricted to 15 cm. The design should be such that the rate of change of radial acceleration is 0.3 m/sec^3. Calculate the length of the transition curve and design speed of vehicles if no lateral pressure is to be exerted on the rails. Also calculate the shift and spiral angle of the transition curve. **(08 Marks)**

8 a. Plot the cross staff survey of a field ABCDFE from the field book measurements given in Fig.Q8(a) below and determine the area of the field. **(10 Marks)**

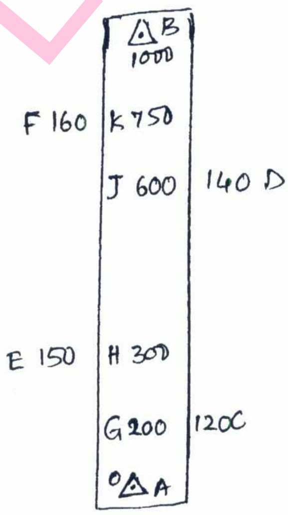

Fig.Q8(a)

b. A road embankment is 30m wide at the top with side slopes of 2 to 1. The ground levels at 100 m intervals along a line AB are as under:

<u>A 170.3</u>, 169.1, 168.5, 168.1 <u>166.5 B</u>

The formation level at A is 178.7m with a uniform falling gradient of 1 in 50 from A to B. Determine the volume of earthwork by prismoidal rule. Assume the ground to be level in cross-section. **(10 Marks)**

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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10CV45

Fourth Semester B.E. Degree Examination, December 2012
Hydraulics and Hydraulic Machines

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART - A

1 a. Distinguish between : i) Geometric and Kinematic similarity ii) Reynolds's and Froude's number iii) Distorted and undistorted model. **(06 Marks)**

b. State and explain Buckingham's π theorem. **(06 Marks)**

c. Using Buckingham's π theorem, obtain an expression for pressure difference ΔP in a pipe of diameter D and height ℓ due to turbulent flow which depends on the velocity V, viscosity μ, density ρ and roughness k. **(08 Marks)**

2 a. Derive the conditions for most economical trapezoidal section. Also show that the most economical trapezoidal section for an open channel is one which has three sides tangential to the semicircle described on the water line. **(12 Marks)**

b. Water is flowing through the circular open channel at the rate of 400 L/s when the channel is having a bed slope of 1 in 9000. Find the diameter of the channel if the depth of flow is 1.25 times radius of channel and Manning's N = 0.015. **(08 Marks)**

3 a. Define the term hydraulic jump. Derive an expression for depth of hydraulic jump in terms of upstream Froude's number. **(10 Marks)**

b. A rectangular channel of bed width 4m is discharging water at the rate of $10m^3/s$. Determine the following : i) Critical depth ii) Minimum specific energy iii) What will be the type of flow in the depth is 0.6m and 2m. **(10 Marks)**

4 a. A 40mm diameter jet having a velocity of 20m/s strikes a flat plate the normal of which is inclined at 30^0 to the axis of jet. If the plate itself is moving with a velocity of 8m/s parallel to itself and is the direction of normal to its surface. Calculate i) Normal force exerted on the plate ii) Work done per second iii) Efficiency of the jet. **(10 Marks)**

b. A jet of water of diameter 25mm strikes a 200mm $\times$ 200mm square plate of uniform thickness with a velocity of 10m/s at the center of the plate which is suspended vertically by a hinge on its top. The weight of the plate is 98.1N. The jet strikes normal to the plate. What force must be applied at the lower edge of the plate so that plate is kept vertical? If the plate is allowed to deflect freely, what will be the inclination of the plate with vertical due to the force exerted by jet of water? **(10 Marks)**

PART - B

5 a. Show that maximum efficiency of the jet striking a series of curved vanes moving in the direction at an angle ϕ, with velocity u is $\eta max = \frac{1+\cos\phi}{2}$. **(08 Marks)**

b. A stationary vane having an inlet angle of zero degree and an outlet angle 25^0 received water at a velocity of 50m/s. Determine the components of force acting on it in the direction of jet and normal to it. Also find the resultant force. If the vane is moving with a velocity 20m/s in the direction of jet, calculate the resultant force, work done and power developed. **(12 Marks)**

6 a. Derive an expression for the work done per second by water on the runner of a Pelton wheel. Hence derive an expression of maximum efficiency of Pelton wheel giving the relationship between the jet speed and bucket speed. **(08 Marks)**

b. A pelton wheel has to be designed for the following data :
Power to be developed = 6000kW, Net head available = 300m, Speed = 550 rpm, ratio of jet diameter to wheel diameter = 1/10 and overall efficiency = 85%. Find the number of jets, diameter of jet, diameter of wheel and quantity of water required. Assume Cv = 0.98 and speed ratio 0.46. **(12 Marks)**

7 a. What is a draft tube? With neat sketch, list the different types of draft tube. **(06 Marks)**

b. What is cavitation? What is its effect on turbine? How it can be avoided in turbines. **(08 Marks)**

c. A Kaplan turbine working under a head of 20m develops 12000 kW. The outer diameter of the runner is 3.5m and inner diameter of the hub is 1.75m. The guide blade angle at the extreme edge of the runner is 35^0. The hydraulic and overall efficiency is 88% and 84% respectively. If velocity of whirl is zero at outlet, determine the runner vane angle at outlet and inlet and also speed of the turbine. **(06 Marks)**

8 a. Differentiate between : i) Pump and Turbine ii) Suction head and delivery head iii) Manometric and overall efficiency iv) Single stage and multistage pumps. **(08 Marks)**

b. What is priming of centrifugal pump? How it is done? **(04 Marks)**

c. A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm works against a total head of 40m. The velocity of flow through the impeller is constant and is equal to 2.5m/s. The vanes are set back at an angle of 40^0 at outlet. If the outer diameter of the impeller is 500mm and width at outlet is 50mm, determine
i) Vane angle at inlet
ii) Work done by impeller on water per second
iii) Manometric efficiency. **(08 Marks)**

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Fourth Semester B.E. Degree Examination, June 2012
Building Planning and Drawing

Time: 4 hrs. Max. Marks:100

Note: ***1. Section-I is compulsory. Answer any TWO questions from Section-II.***
2. Suitable data may be assumed wherever necessary.

SECTION – I (Compulsory)

1 The line diagram of a residential building is given in Fig.Q.1. Draw to a scale of 1:100.

a. Plan at sill level. **(25 Marks)**
b. Front elevation. **(15 Marks)**
c. Section on x – x. **(15 Marks)**
d. Schedule of openings. **(05 Marks)**

Note: All load bearing walls are of 230mm thick, BBM built on SSM foundation. Roof is RCC and the roof height is 3.0m from floor finish. Lintel level is 2.1m above the plinth level. Assume suitable size for openings.

SECTION – II

2 Draw a cross section and plan of a RCC dog legged stair for a building having the following particulars:

a. Clear size of stair hall = 2.5m × 4.5m.
b. Width of landing = 1.2m.
c. Width of each flight = 1.2m.
d. Rise = 150mm and tread = 300mm.
e. Thickness of waist slab = 150mm.
f. Height of floor = 3.6m. **(20 Marks)**

3 It is proposed to construct a primary school building for rural area with a plinth area requirement of about 300m^2 on a site measuring 70m × 100m; of which 100m side faces the road. Provide the following requirements:

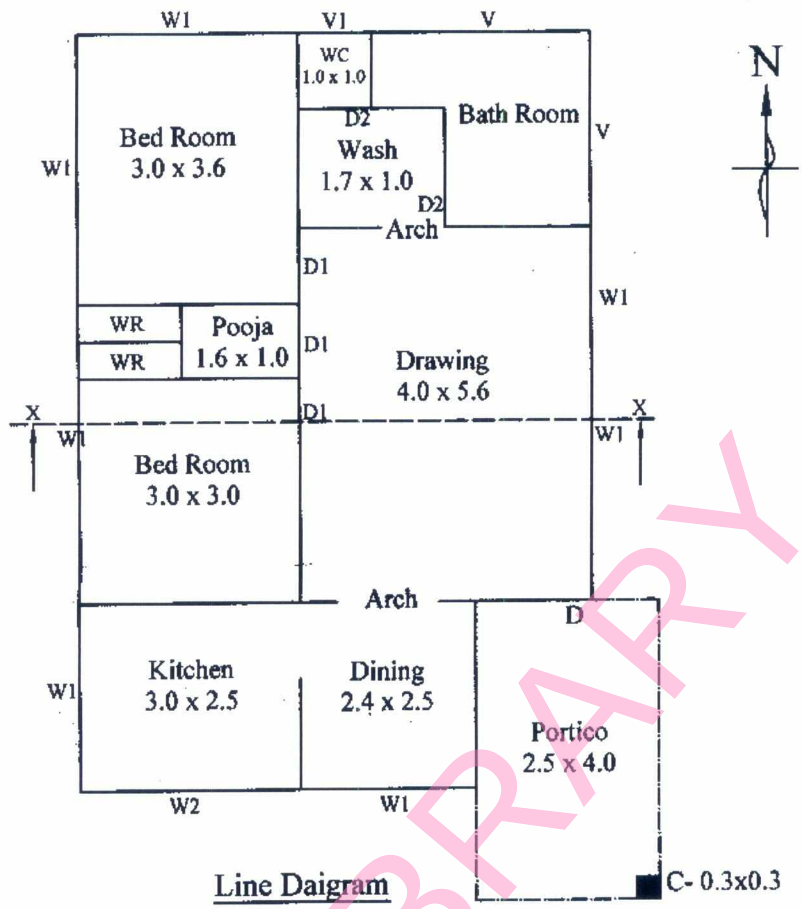
a. Head Master's room.
b. Staff room with toilets.
c. 06 numbers of class rooms
d. Library.
e. Sports room.
f. Toilets for students.

The minimum set backs to be provided as per bye-laws. Prepare a line diagram with dimensions to a scale 1:100. **(20 Marks)**

4 Prepare a bubble diagram (connectivity diagram) and develop a line diagram for a canteen building of an engineering college and requirements for the building are:

a. Dining area for boys and girls separately.
b. Dining area for staff.
c. Kitchen.
d. Stores for kitchen.
e. Utilities attached to kitchen.
f. Students strength of the college is 2500. **(20 Marks)**

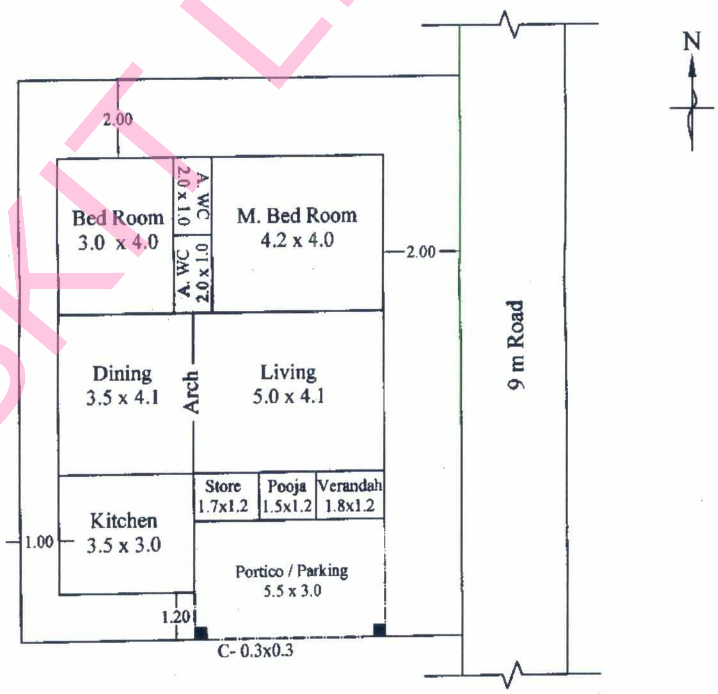
5 The line diagram of a residential building is shown in Fig.Q.5. Prepare water supply, sanitary and electrical layout plans with usual notations to a scale of 1:50. **(20 Marks)**



Line Daigram

All dimensions are in meters

Fig.Q.1



Line Daigram

All dimensions are in meters

Fig.Q.5

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MATDIP401

Fourth Semester B.E. Degree Examination, December 2012

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Prove that the angle between two lines whose direction cosines are (ℓ_1, m_1, n_1) and (ℓ_2, m_2, n_2) is $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$. **(06 Marks)**

b. Find the projection of the line AB on CD where A = (1, 3, 5), B = (6, 4, 3), C = (2, -1, 4) and D = (0, 1, 5). **(07 Marks)**

c. Find the angle between any two diagonals of cube. **(07 Marks)**

2 a. Find the equation of the plane passing through the points (3, 1, 2) and (3, 4, 4) and perpendicular to $5x + y + 4z = 0$. **(06 Marks)**

b. Show that the points (0, -1, 0), (2, 1, -1), (1, 1, 1) and (3, 3, 0) are coplanar. **(07 Marks)**

c. Find the equation of the plane through the points (1, 0, -1), (3, 2, 2) and parallel to the line $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-2}{3}$. **(07 Marks)**

3 a. Find the value of λ such that the vectors $\lambda i + j + 2k$, $2i - 3j + 4k$ and $i + 2j - k$ are coplanar. **(06 Marks)**

b. If $\vec{a} = 4i + 2j - k$, $\vec{b} = 2i - j$ and $\vec{c} = j - 3k$, find (i) $(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c})$, (ii) $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})$. **(07 Marks)**

c. Find the cosine and sine of the angle between the vectors $2i - j + 3k$ and $i - 2j + 2k$. **(07 Marks)**

4 a. Find the components of velocity and acceleration at t = 2 on the curve, $\vec{r} = (t^2+1)i + (4t-3)j + (2t^2-6t)k$ in the direction of $i + 2j + 2k$. **(06 Marks)**

b. Find the angle between the tangents to the curve $\vec{r} = \left\{t - \frac{t^3}{3}\right\}i + t^2 j + \left\{t + \frac{t^3}{3}\right\}k$ at $t = \pm 3$. **(07 Marks)**

c. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2i - j - 2k$. **(07 Marks)**

5 a. If $\vec{F} = \nabla(xy^3z^2)$, find div $\vec{F}$ and curl $\vec{F}$ at the point (1, -1, 1). **(06 Marks)**

b. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\nabla^2(\log r) = \frac{1}{r^2}$ where $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Find Laplace transform of $(2t + 3)^2$. **(05 Marks)**

b. Find Laplace transform of $e^{2t}\cos 3t$. **(05 Marks)**

c. Find $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$. **(05 Marks)**

d. Using Laplace transform, evaluate $\int_0^\infty e^{-2t} t \cos t \, dt$. **(05 Marks)**

7 a. Find inverse Laplace transform of $\frac{s}{s^2 + 4s + 13}$. **(06 Marks)**

b. Find $L^{-1}\left\{\frac{1}{(s^2 + 3s + 2)(s + 3)}\right\}$. **(07 Marks)**

c. Find $L^{-1}\left\{\log\left(\frac{s^2 + 1}{s^2 + s}\right)\right\}$. **(07 Marks)**

8 a. Solve the differential equation $y'' + 4y' + 3y = e^{-t}$ with $y(0) = 1$ and $y'(0) = 1$ by using Laplace transforms. **(10 Marks)**

b. Solve by using Laplace transforms $\frac{dx}{dt} - 2y = \cos 2t$, $\frac{dy}{dt} + 2x = \sin 2t$ with $x = 1$, $y = 0$ at $t = 0$. **(10 Marks)**

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